Session 7
Climb and Descent Performance

1.0 Definitions

Rate-of-climb - The straight-up vertical velocity, measured in feet per second. The abbreviation for rate-of-climb is *RC*.

Climb angle - The number of degrees between the horizon and the flightpath of the aircraft. The abbreviation for climb angle is the Greek letter gamma, γ.

2.0 Introduction

The climb performance of an aircraft is an important safety of flight consideration as it determines the capability of the aircraft to clear an obstacle after takeoff, enroute terrain avoidance, and go-around capability from an aborted landing. Due to the safety of flight considerations, the Federal Aviation Administration (F.A.A.) has minimum angle of climb criteria for those flight modes close to the ground such as takeoff and landing and also for the engine out (emergency) phase of flight for multi-engine aircraft.

Figure 7.1 Takeoff

3.0 Theory

From basic trigonometry,

$$\sin \gamma = RC/V$$

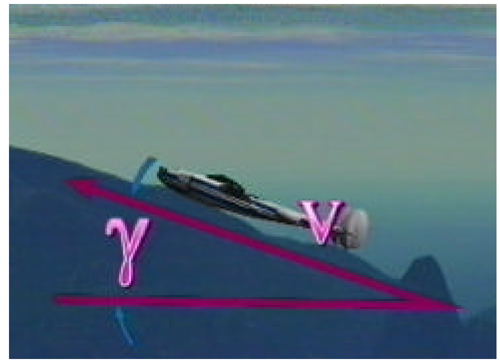

Figure 7.2 Climb Angle

Video Example: aircraft is flying at 200 knots [or 338 ft/sec]. If the climb <u>angle</u> is 10 degrees, then $RC = V \sin g = 338[ft/sec] \sin 10° = 58.7[ft/sec]$

3.1 Simplifying Assumptions

The first simplifying assumption is used only at the basic level to illustrate the principle factors in climb performance: **The aircraft's angle of attack is small.**

Figure 7.3 Angle of Attack is Small

NOTE:

A non-trivial angle of attack complicates the equations a bit, but is used for all accurate analysis: Summing up the forces <u>along the direction of the flightpath</u>, the tilt of the thrust line must be included:

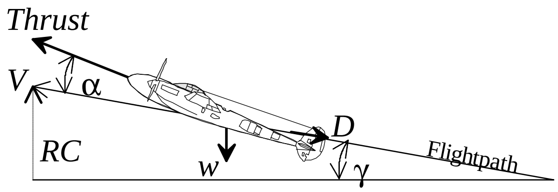

Figure 7.4 $\Sigma F = T \cos \alpha - D - w \sin \gamma$

Along the flightpath:

$$\Sigma F = T \cos \alpha - D - w \sin \gamma$$

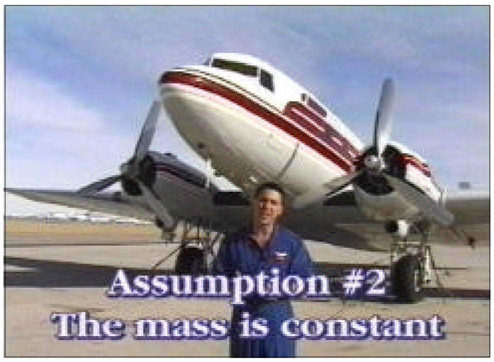

Figure 7.5 Mass is Constant

The next simpifying assumption: **The aircraft's mass is constant** is quite reasonable for all propeller aircraft and most jets. This assumption simplifies the equation to:

$$T - D - w \sin \gamma = m\, \Delta V/\Delta t$$

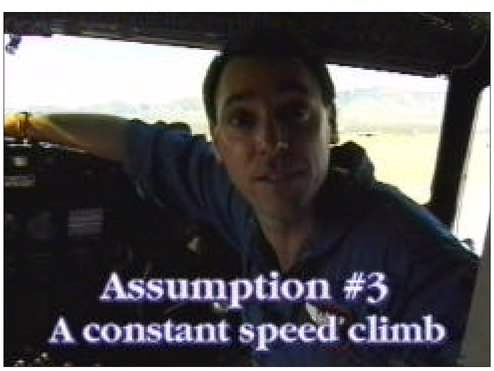

Figure 7.6 Airspeed is Constant

The equations used if the "constant mass" assumption is not valid, can be found in the "energy method" section of this guide. This section also shows how to look at a plane's ability to climb and accelerate at the same time. To avoid complication, the video made a third simplifying assumption: **The aircraft is climbing at a constant speed** so that $\Delta V/\Delta t = 0$.

With no mass or velocity change, the sum of the forces is zero:

$$T - D - w \sin \gamma = 0 \quad \text{or} \quad [T - D]/w = \sin \gamma.$$

The numerator $[T - D]$ is called the excess thrust because it is the extra thrust available after the aircraft's drag is overcome.

The video stated that there are several ways to measure the drag; it repeated the method in Session 4 with the glider and calculated its drag at a particular airspeed. This gliding test works great on sailplanes, but not on big airplanes because it isn't safe to shut down all the engines. In this case, flight testers use knowledge of engine thrust which is opposite to the drag force. The thrust prediction provided by the manufacturer is usually very complicated, but a simplified version was used in the video.

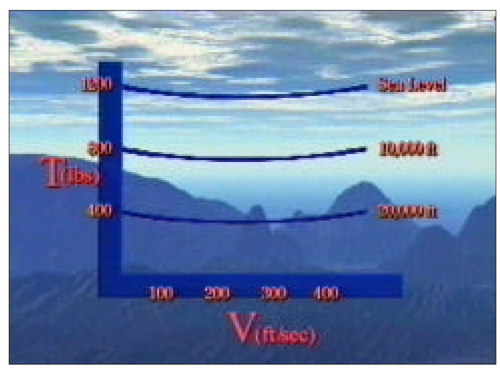

Figure 7.7 Thrust Prediction

At any one altitude, the thrust changes just a little as the airspeed increases. For any given airspeed, the thrust gets smaller as altitude increases. For the example in the video, the flight condition was 300 ft/sec and 5,000 feet altitude. The thrust was 1000 lbs., the drag was 400 lbs. and the aircraft weight was 4200lbs. Putting all of this together gave:

$$[1000 \text{ lbs.} - 400 \text{ lbs.}]/4200 \text{ lbs.} = \sin \gamma$$

$$\text{or} \quad 0.1428 = \sin \gamma$$

solving for γ gave $\gamma = \sin^{-1} 0.1428 = 8.2^{\circ}$

The above relation for sin γ can be inserted into the rate-of-climb equation ($RC = V \sin \gamma$) to give

Climb Rate Equation:

$$RC = V\frac{[T-D]}{w}$$

so, RC = 300 *ft/sec* (0.1428) = 42.8 *ft/sec.*

3.2 Climb Rate vs Velocity

Drag increases with the square of velocity. Compare the drag to the engine thrust available at sea level. The vertical distance between the two curves is the excess thrust, $F - D$. As the airspeed increases, the excess thrust gets smaller and smaller. At very low speeds there is a lot of excess thrust, but the velocity is small, so the climb rate is moderate. At medium speeds, there is not quite as much excess thrust, but multiplying it by the higher speed gives a good climb rate. Finally, at high speed, the excess thrust is very small. Even though the speed is high, the product of the two yields a poor climb rate. This should sound reasonable since most of the available thrust is needed just to overcome the drag, leaving little excess thrust for climbing.

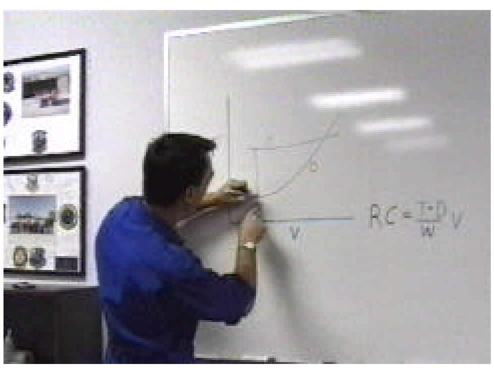

Figure 7.8 Execss Thrust

To determine the altitude effect on climb performance, first go back to the engine chart. Since the air is less dense at high altitude, the maximum thrust of the engine will also be less. At 23,000 feet where the density is half of that at sea level, the thrust will also be about half of the sea-level value. Of course, the profile drag will also be about half of the sea-level value. Since both the thrust and drag are reduced by 50%, then the excess thrust reduction will be the same. Finally, the climb rate and angle will be about half of the sea level climb capability.

4.0 Power Method

Climb performance is directly related to the excess **power** available. This is the difference between the power required for level flight and the power available from the propulsion system at a particular airspeed and density altitude. The video showed that climb performance is a function of excess thrust available, which is also true. The connection between the two is quite simple: thrust times velocity equals power ($P = TV$).

The video showed that climb rate is $RC = V\frac{[T-D]}{w}$ where $\frac{[T-D]}{w}$ is the specific excess thrust. Climb rate is velocity *times* specific excess thrust or simply specific excess power. In a similar fashion, since the sine of the climb angle is the specific excess thrust, then it is also the specific excess power *divided* by the speed.

$$\sin c = \frac{F-D}{w} = \frac{(F-D)V}{w}\,\frac{1}{V}$$

Figure 7.7 illustrates this for both a jet and a propeller aircraft. The excess *power* can be used to either climb or accelerate the aircraft; therefore, knowledge of the excess power available at each altitude and airspeed will define the aircraft climb performance, level acceleration performance, or any combination of the two. Conversely, measurement of the climb and/or acceleration performance of an aircraft will define the specific excess power.

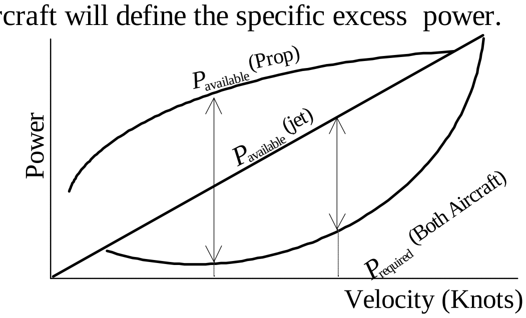

Figure 7.9 Maximum Rate of Climb, Prop and Jet

5.0 Energy Method for Climb Performance

If an energy approach is used where the total energy of an aircraft is expressed as the sum of the potential and kinetic energy, basic physics states that a change in energy requires that work be done (Figure 7.8).

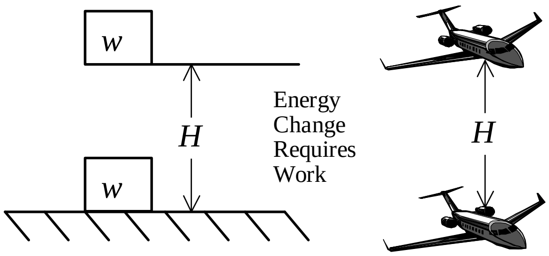

Figure 7.10 Power Available - Physics

The rate of change of energy requires the application of power which is the work done per time interval. Since the total energy of the aircraft is changed by the excess power available then:

Excess Power Available

$$= \frac{d}{dt}\,[\text{Potential Energy} + \text{Kinetic Energy}]$$

$$= \frac{d}{dt}\left[wH + \frac{w}{2g}V^2\right]$$

$$= w\frac{dH}{dt} + H\frac{dw}{dt} + V\frac{w}{g}\frac{dV}{dt} + \frac{V^2}{2g}\frac{dw}{dt}$$

This complete equation is needed for rockets and aircraft with extreme fuel flow rates such as the F-22 in full afterburner. For most general aviation commercial transport aircraft however, the rate of change of weight $\frac{ds}{dt}$ is very small and can be neglected with the result that:

$$\text{Excess Power} = X_sP = \left[w\frac{dH}{dt} + \frac{wV}{g}\frac{dV}{dt}\right]$$

where $\frac{dH}{dt}$ is the time rate of change of altitude, $\frac{dV}{dt}$ is the time rate of change of true velocity in ft/sec and V is the velocity in ft/sec The unit of power is ft-lb per second. Since an aircraft has a fixed amount of excess power at any given flight condition, this equation can be used to show the plane's ability to climb at constant velocity, accelerate at constant altitude, or some combination of both climb and acceleration.

To **measure** the excess power available at any altitude, it is necessary to measure the rate of climb, $\frac{dH}{dt}$ and the flight path acceleration, $\frac{dV}{dt}$. The common technique is to keep one of the variables constant and measure the rate of change of the other. The excess power can thus be measured by the rate of climb (sawtooth climb) test or by the level acceleration. The term "sawtooth" climb is used to describe a series of climbs where the pilot climbs through an altitude band at some constant airspeed (so that $\frac{dV}{dt} = 0$), then descends, then repeats the climb at another constant airspeed and so forth.

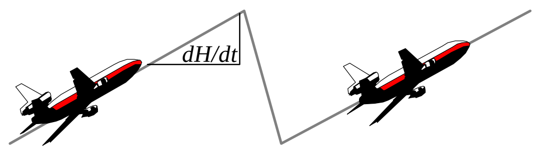

Figure 7.11 Sawtooth Climb Technique

During each climb, the pilot records the airspeed and weight and times the ascent with a stopwatch to get the climb rate. The results of a series of sawtooth climb tests can be plotted as shown in Figure 7.11.

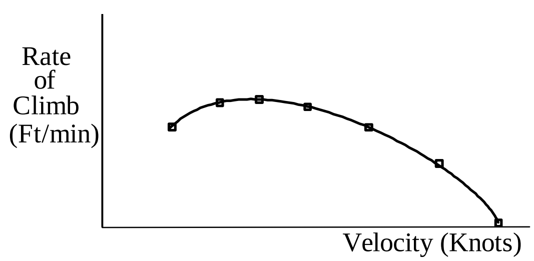

Figure 7.12 Plot of Sawtooth Climb Data

6.0 Data Analysis

When the rate of climb data is taken at different altitudes, corrected it can be presented as seen in Figure 7.11. The top of each curve gives the maximum rate of climb at particular altitudes and the speed that must be held to obtain that maximum rate of climb. The tangents from the origin give the velocities for the maximum angle of climb. The speeds for maximum angle of climb and maximum rate of climb are defined as V_x and V_y respectively. A typical plot of the variation of V_x and V_y with altitude is given in Figure 7.12 where it can be seen that at the absolute ceiling of the aircraft $V_x = V_y$.

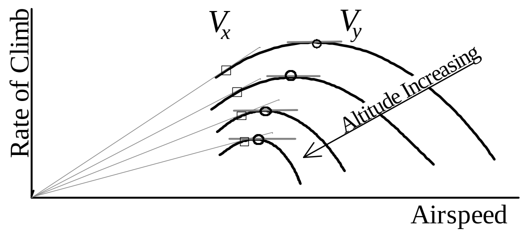

Figure 7.13 Climb Data as a Function of Altitude

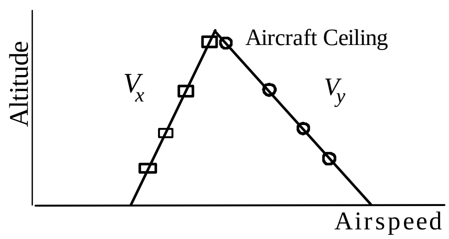

Figure 7.14 Variation of V_x and V_y with Altitude

7.0 Descents

This very same rate and angle of climb equations also work for an airplane that is descending at a constant airspeed. All the pilot has to do is decrease the thrust until it is less than the drag. This means that the excess thrust is a *negative* value. Substituting a negative value into the climb rate equation means the aircraft is descending. If the excess thrust is a *large* negative value, then the airplane will descend faster. This concept was shown with the glider in Session 4 (although the intent of that video segment was to illustrate the change in drag). The brakes added more drag thereby making a more negative excess thrust.

8.0 Summary

Assuming a small angle of attack,

$$RC = V \sin \gamma$$

Then, starting from Newton's second law and assuming a constant mass and velocity, simple calculations give the equation for predicting climb angle. Note that the climb angle is directly related to the *specific excess thrust*.

$$[T - D]/w = \sin \gamma$$

And finally, combining these two gave the rate of climb equation. Note that the climb rate is directly related to the *specific excess power*.

$$RC = V[T - D]/w$$

Examples 5.1 and 5.2 in the textbook give further illustrations of these lessons.

9.0 Measures of Performance

1 What happens to the climb rate and climb angle of an aircraft if the weight increases?

ANSWER:

Since $RC = \dfrac{V(T-D)}{w}$ then a larger weight **reduces** climb rate. Similar result of climb angle.

2 Why does the climb rate decrease at high altitudes?

ANSWER:

Because less thrust is available at low air density.

3 What climb measurement is directly related to specific excess thrust?

ANSWER:

Specific Excess Thrust = $\dfrac{T-D}{w} = \sin c$ it's related to climb **angle**.

4 For the simplified math presented in the video, what were the assumptions?

ANSWER:

Negligible angle of attack, weight change and velocity change